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1993 J. Phys. A: Math. Gen. 26 1063

(http://iopscience.iop.org/0305-4470/26/5/025)

J. Phys. A: Math. Gen. 26 (1993) 1063–1066. Printed in the UK

Monte Carlo study of the six-dimensional Ising spin glass

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Received 6 August 1992

Abstract. We study the Ising spin glass with a $\pm J$ distribution in six dimensions by Monte Carlo simulations. Our results for the transition temperature agree very well with the value from series expansions. The exponents are consistent with mean-field values modified by logarithmic corrections, as expected.

1. Introduction

The theory of spin glasses (for a review of spin glasses see [1]) is, by now, fairly convincing in its prediction [2–6] that there is a transition in three dimensions for systems with Ising symmetry. Another prediction of the theory is that the upper critical dimension, d_u, above which mean-field theory should apply, is equal to six, [7], which is different from ferromagnetic systems for which $d_u = 4$. Evidence for mean-field behaviour setting in at $d = 6$ has been seen from series expansions [8, 9]. Since the claim that $d_u = 6$ is one of the main predictions of spin glass theory, it seems useful to provide further evidence for it by other techniques. Here we use Monte Carlo simulations to show that the six-dimensional Ising spin glass with $\pm J$ interactions does have essentially mean-field behaviour. Since we are working *precisely* at the upper critical dimension, logarithmic corrections to mean-field theory are expected and our results are consistent with this.

2. The model

The Hamiltonian is given by

$$\mathcal{H} = -\sum_{i,j} J_{i,j} S_i S_j \tag{1}$$

where the nearest-neighbour interactions, $J_{i,j}$, are independent random variables which take values ± 1 with equal probability. The Ising spins, S_i, take values ± 1 and lie on a six-dimensional hypercubic lattice with periodic boundary conditions with $N = L^6$ sites. Our results are for $2 \leqslant L \leqslant 6$.

Tests for equilibration are done by simultaneously studying two identical replicas for each bond configuration, as described elsewhere [10]. The number of Monte Carlo sweeps varied from 640 for $L = 2$ to 2560 for $L = 6$. We averaged over a large number of bond configurations, N_c, where $N_c = 8000$ for $L = 2$ and $N_c = 5330$ for $L = 6$. The simulations used standard Monte Carlo techniques and took a total of order 2000 h of CPU time distributed among Sun 4 and IBM RS6000 workstations.

0305-4470/93/051063+04$07.50

We used the histogram technique [11], by which a simulation at one temperature can be used to give expectation values at other neighbouring temperatures. Our results are analysed by standard finite-size scaling techniques [10], which we briefly summarize again here. After t_0 sweeps for equilibration, one measures the overlap, $q(t)$, between the configurations of the two replicas t sweeps later,

$$q(t) = \frac{1}{N}\sum_{i=1}^{N} S_i^{(1)}(t_0+t)S_i^{(2)}(t_0+t). \tag{2}$$

Moments of $q(t)$ are then calculated, and the analysis focuses particularly on the spin glass susceptibility,

$$\chi_{\rm SG} = N\langle q^2\rangle \tag{3}$$

and the dimensionless ratio,

$$g = \frac{1}{2}\left(3 - \frac{\langle q^4\rangle}{\langle q^2\rangle^2}\right) \tag{4}$$

where the average $\langle\cdots\rangle$ denotes both a time average for a given set of bonds and an average over bond configurations. Since g is dimensionless it has the simple finite-size scaling form

$$g = \tilde{g}(L^{1/\nu}(T - T_{\rm c})) \tag{5}$$

where ν is the correlation length exponent. The finite-size scaling form for $\chi_{\rm SG}$ is

$$\chi_{\rm SG} = L^{2-\eta}\tilde{\chi}_{\rm SG}(L^{1/\nu}(T - T_{\rm c})) \tag{6}$$

where η describes the power-law decay of the correlations at $T_{\rm c}$ and is related to the exponent γ, describing the divergence of $\chi_{\rm SG}$ with temperature, by

$$\gamma = (2-\eta)\nu. \tag{7}$$

The mean-field values are

$$\eta = 0 \tag{8}$$

$$\nu = \tfrac{1}{2}. \tag{9}$$

There are three unknown quantities in this analysis, $T_{\rm c}$, ν and η. These are determined *independently* of each other as follows. First, it follows from (5) that the curves for g against T for different sizes intersect at $T_{\rm c}$. Having determined $T_{\rm c}$ from this intersection, we can obtain ν from the derivative of g with respect to T at $T = T_{\rm c}$ since, according to (5), $\mathrm{d}g/\mathrm{d}T|_{T=T_{\rm c}} \sim L^{1/\nu}$. Finally, η is determined from the dependence of $\chi_{\rm SG}$ with L at $T_{\rm c}$, namely $\chi_{\rm SG}(T_{\rm c}) \sim L^{2-\eta}$, which follows from (6).

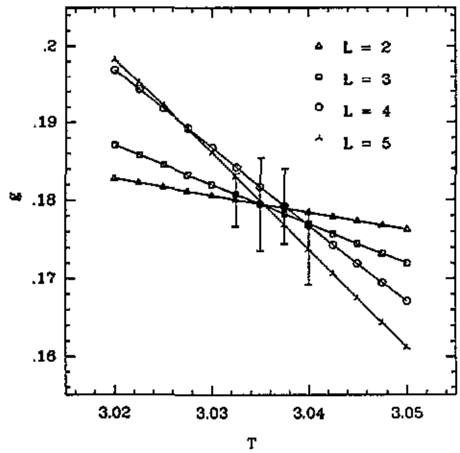

Figure 1. The dimensionless ratio of moments, g, defined in (4), is plotted against T for sizes from $L = 2$ to $L = 5$. The curves intersect at the transition temperature T_c. From this data we estimate that $T_c = 3.035 \pm 0.01$. Note the greatly expanded scales on both axes.

3. Results

In figure 1 we plot data for g against T for sizes between 2 and 5. Note that both the horizontal and vertical scales are greatly expanded. For each size, all the data came from one long run and results for the different temperatures were obtained by the histogram method [11]. From the intersection of the data we estimate $T_c = 3.035 \pm 0.01$. This compares extremely well with the estimate of 3.027 ± 0.005 from a fifteen-term series expansion [9].

The values of $\mathrm{d}g/\mathrm{d}T$ at $T \simeq T_c = 3.0325$ are plotted against L in a double logarithmic plot in figure 2. The slope of the fit, which should equal $1/\nu$, is equal to 2.02 ± 0.08. This shows that ν is well given by the mean-field prediction $\nu = 1/2$. There is no sign of logarithmic corrections, which would appear as curvature in the plot.

Evidence for logarithmic corrections does, however, show up in the results for χ_{SG} at $T = 3.0325$ presented in figure 3. The data clearly do not lie on a straight line and, furthermore, the average slope is different from the mean-field value of two, shown by the dotted curve. We have tried to analyse this data by assuming a power law with logarithmic corrections [8, 9]:

$$\chi_{SG}(T_c) = AL^{2-\eta}\,[\log(L/L_0)]^{\theta}\,. \tag{10}$$

Because our data are over a limited range of L we are unable to determine unambiguously all four parameters in this fit. Based on our finding that ν is given by the mean-field value, we have fitted our data to (10) *assuming* that η has the mean-field value of 0. The fit is shown in figure 3 and corresponds to parameter values $\theta = 0.64$ and $L_0 = 1.01$. Thus, while we have not *proved* that η is given by its mean-field value, we have shown that the form in (10) works very well with this assumption and very reasonable values for the other parameters.

Logarithmic corrections *are* expected at the upper critical dimension and the correction to χ_{SG} has been evaluated in the thermodynamic limit, $L \gg \xi$ [8]. However, the form of the corrections in the finite-size regime, $\xi \gg L$, which are of interest here, have not, to our knowledge, been elucidated even for the simpler case of an Ising ferromagnet. The only result of which we are aware is for the ferromagnet with an infinite number of spin components [12]. It would clearly be desirable to determine *analytically* the logarithmic corrections in the finite-size regime for the spin glass, to compare with the fits obtained from our numerical data.

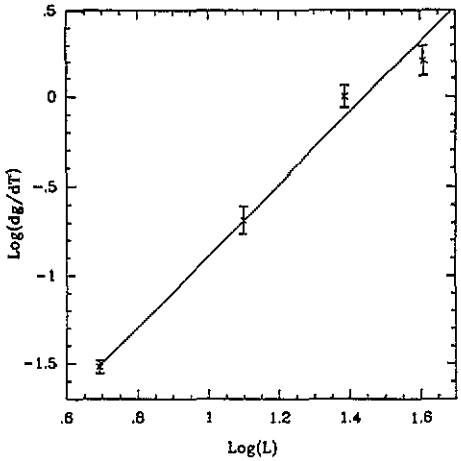

Figure 2. A log–log plot of dg/dT against L at $T = 3.0325$ for sizes from $L = 2$ to $L = 5$. The slope of the fit, which should equal $1/\nu$ from (5), is equal to 2.02 ± 0.08, consistent with ν having the mean-field value of $\frac{1}{2}$.

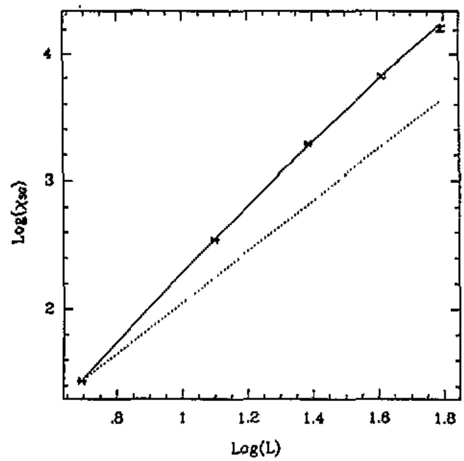

Figure 3. A log–log plot of χ_{SG} against L at $T = 3.0325$ for sizes from $L = 2$ to $L = 6$. The data are curved showing that the simple power law expected from (6), which gives a constant slope of $2 - \eta$, does not hold. The full curve is a fit to a modified form with logarithmic corrections shown in in (10), with η set to zero. The dotted curve has a slope of two, corresponding to the mean-field result, $\eta = 0$, without logarithmic corrections.

4. Conclusions

We have shown that the six-dimensional Ising spin glass has a mean-field transition modified by logarithmic corrections, in agreement with conventional expectations. It would be desirable to determine analytically the form of the logarithmic corrections in the finite-size regime.

Acknowledgment

This work is supported by the NSF through grant number DMR 91-11576.

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